DISSIPATIVE INSTABILITY OF A NONISOTHERMAL ELECTRICALLY CONDUCTING FLOW BETWEEN PARALLEL PLATES IN A TRANSVERSE MAGNETIC FIELD

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Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 8, No. 5, pp. 21–28, 1967

Problems of dissipative instability (in particular, overheating) in magnetohydrodynamics has been studied in [1–6]. The Leontovich mechanism of overheating instability is explained in [1] by the example of a stationary homogeneous plasma in a strong magnetic field along which current flows. The rate of buildup of perturbations is estimated in [2] to explain the effect of overheating instability on the operation of an MHD generator. The effect of inhomogeneity in the temperature field and in the boundaries of the region on the formation of this instability has been studied by the example of discharge in a stationary medium in the absence of a magnetic field [3]. Certain cases of overheating instability in magnetohydrodynamics are considered in [4, 5], where it is shown that it can be aperiodic as well as oscillatory (Alfven and acoustic waves). Finally, the hydrodynamic and overheating branches of instability in the case of nonisothermal plasma flow in a plane MHD channel was investigated in [6]. But the overheating instability was examined without allowance for the dependence of the viscosity and thermal-conductivity coefficients on temperature in the limiting case $S \ll R_m \ll 1$ and only for small perturbation wavelengths.

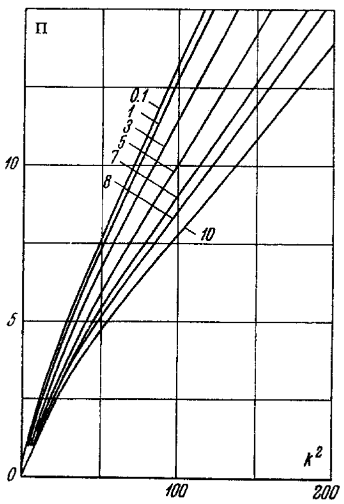

Fig. 1

The development of shortwave perturbations is studied below with allowance for viscosity and thermal conductivity and for a wider range of conditions $A \ll 1$. Overheating instability over the entire range of wavelengths for the case considered in [6] is also studied.

1. Steady nonisothermal plasma flow in a plane MHD channel was studied in [7,8]. Let the x axis be directed along the flow; let the y axis be directed along the external homogeneous magnetic field B, and let the z axis be directed along the constant electric field E. The channel is bounded by dielectric plates at $y = \pm l$. It is assumed that the transport coefficients are power functions of temperature

$$\sigma = \sigma_0 (T / T_0)^{\alpha}, \quad \varkappa = \varkappa_0 (T / T_0)^{\beta}, \quad \eta = \eta_0 (T / T_0)^{\gamma}, \qquad (1.1)$$

where σ_0, $\varkappa_0$, and η_0 are the electrical-conductivity, thermal-conductivity, and viscosity coefficients at the wall temperature T_0. Moreover, it is assumed that the applied fields do not upset the isotropy of the medium's properties. Assuming that in the steady state all values are functions of y, we can formulate a nonlinear boundary-value problem whose solution will give the distribution of velocity, temperature, and other values between the plates. This was done in [8] and the results will be used in solving the eigenvalue problem for perturbation wavelengths that are comparable with or greater than the distance between the plates. The velocity, temperature, and current-density distributions are functions of the six similarity parameters:

$$\alpha, \ \beta, \ \gamma, \ M = Bl \left(\frac{\sigma_0}{\eta_0}\right)^{1/2},$$

$$K = -\frac{E\eta_0}{l^2 pB} \ \text{ or } \ J = \frac{IB}{2pl}, \quad N = \frac{l^4 p^2}{\varkappa_0 \eta_0 T_0}, \qquad (1.2)$$

where p is the pressure gradient along the flow and I is the current per unit length of channel. Besides these parameters, the induced magnetic field B_x is also a function of the magnetic Reynolds number R_m.

Let us formulate the problem of the stability of the described steady state with respect to two-dimensional perturbations which are of particular interest in

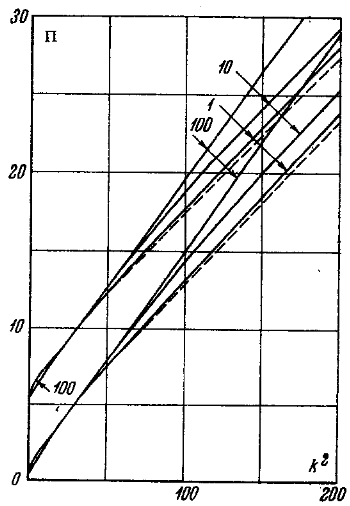

Fig. 2

view of the flow geometry. The perturbations u', v', B_x', B_y', j_z', and T', which are functions of x, y, and z, can exist in this case. A general system of equations for the perturbations, which simultaneously takes into account the effect on stability of hydrodynamic, electrodynamic, and thermal effects, is given in [6].

Below, we shall limit our examination to the case in which velocity perturbations are small and the development of instability is determined by the dynamics of current-density and temperature perturbations. This is the case when the conditions [6]

$$A = \frac{B^2}{\mu\rho v^{*2}} \ll 1, \qquad \frac{\Upsilon}{R} = \frac{\Upsilon\eta_0}{\rho l v^*} \ll 1 \tag{1.3}$$

are satisfied.

Here A is the Alfven number, R is the Reynolds number, μ is the permeability, ρ is the density, and v^* is the characteristic flow velocity. It is convenient to introduce the stream flow function for magnetic-field perturbations

$$B_x' = \partial\varphi/\partial y, \quad B_y' = -\partial\varphi/\partial x, \quad \mu j_z' = -\Delta\varphi\,. \tag{1.4}$$

If we let θ denote the temperature perturbation, it is easy to write the initial system of equations

$$\frac{\partial\varphi}{\partial t} + v\,\frac{\partial\varphi}{\partial x} = \frac{1}{\mu\sigma}\Delta\varphi + \frac{\alpha j}{\sigma T}\theta, \tag{1.5}$$

$$\rho c\left(\frac{\partial\theta}{\partial t} + v\,\frac{\partial\theta}{\partial x}\right) = \varkappa\Delta\theta + 2\beta\varkappa(\ln T)'\,\frac{\partial\theta}{\partial y} +$$

$$+\left[-\beta\varkappa(\ln T)'^2 + (\gamma-\beta)\frac{\eta v'^2}{T} -\right.$$

$$\left.-(\alpha+\beta)\frac{j^2}{\sigma T}\right]\theta - 2\frac{j}{\mu\sigma}\Delta\varphi\,, \tag{1.6}$$

where v, T, and j are the velocity, temperature, and current density in an unperturbed flow; c is the specific heat; σ, $\varkappa$, and η are defined according to (1.1);

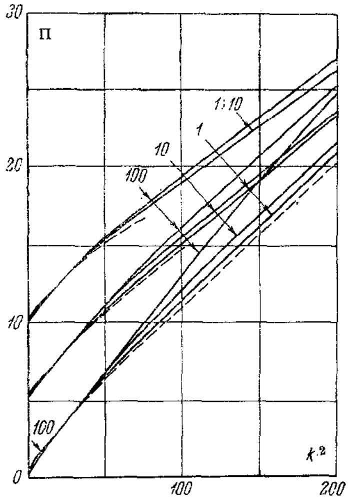

Fig. 3

the primes indicate differentiation with respect to y. Let us introduce the following dimensionless values (primes will be omitted below):

$$t' = \frac{tv^*}{l}, \qquad x' = \frac{x}{l}, \qquad y' = \frac{y}{l},$$

$$\varphi' = \frac{\varphi}{Bl}, \qquad v' = \frac{v}{v^*},$$

$$j' = \frac{j}{\sigma_0 v^* B}, \quad T' = \frac{T}{T_0}, \quad \theta' = \frac{\theta}{T_0},$$

$$\sigma' = \frac{\sigma}{\sigma_0}, \qquad \varkappa' = \frac{\varkappa}{\varkappa_0}, \qquad \eta' = \frac{\eta}{\eta_0}. \tag{1.7}$$

Then we can rewrite Eqs. (1.5) and (1.6) in dimensionless form

$$\frac{\partial\varphi}{\partial t} + v\,\frac{\partial\varphi}{\partial x} = \frac{1}{R_m}\frac{1}{\sigma}\Delta\varphi + \frac{\alpha j}{\sigma T}\theta\,, \tag{1.8}$$

$$\frac{\partial\theta}{\partial t} + v\,\frac{\partial\theta}{\partial x} = \frac{1}{P}\varkappa\Delta\theta + \frac{2\beta}{P}\varkappa(\ln T)'\frac{\partial\theta}{\partial y} +$$

$$+\left[-\frac{\beta}{P}\varkappa(\ln T)'^2 + (\gamma-\beta)\frac{Q}{P}\frac{\eta v'^2}{T} -\right.$$

$$\left.-(\alpha+\beta)S\frac{Q}{N_P}\frac{j^2}{\sigma T}\right]\theta - 2A\,\frac{Q}{N_P}\frac{j}{\sigma}\Delta\varphi$$

$$\left(R_m = \mu\sigma_0 v^* l,\ P = \frac{l v^* \rho c}{\varkappa_0},\right.$$

$$\left.Q = \frac{\eta_0 v^{*2}}{\varkappa_0 T_0},\ S = AR_m,\ N_P = \frac{\eta_0 c}{\varkappa_0}\right) \tag{1.9}$$

where R_m is the magnetic Reynolds number, P is the Peclet number, Q is a thermal parameter similar to (1.2), S is the hydromagnetic-interaction parameter,

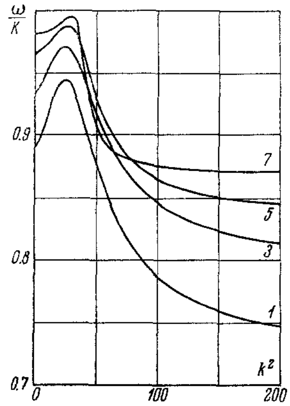

Fig. 4

and N_P is the Prandtl number. Since the coefficients of (1.8) and (1.9) are functions only of y, the system is solved in the form

$$\varphi = \varphi(y)\exp(ikx - i\omega t) \qquad (\omega = \omega_r + i\omega_i)\,, \tag{1.10}$$

where k is the dimensionless wave number and ω is the dimensionless oscillation frequency. Equations (1.8) and (1.9) must be solved under the amplitude boundary conditions

$$\theta(\pm 1) = 0, \qquad (\varphi'/\varphi)_{\pm 1} = \mp k\,. \tag{1.11}$$

2. Let us study the behavior of "shortwave" perturbations when the characteristic dimension for the variation in the amplitude of perturbation λ_y is small in comparison with the scale for unperturbed values l. If we represent the amplitudes in quasi-classical form

$$\varphi(y) = \exp\left(i\int^{y} k_y dy\right), \tag{2.1}$$

where k_y is a slowly varying function of y, the case in question can be defined by the condition $|k_y| \gg 1$.

Let us assume, first of all, that the viscosity and thermal-conductivity coefficients are approximately constant. This occurs, for example, with the dense low-temperature plasma of MHD generators, which is characterized by a strong conductivity-temperature dependence (approximated by the formula $\sigma \approx \exp(-A/T)$ or $\sigma \approx T^{10-13}$). If we let $\beta = \gamma = 0$, if we eliminate θ from (1.9) by using (1.8), and if we drop the terms $\sim k_y^{-1}$, we obtain

$$\varphi^{IV} - [2k^2 + (\sigma R_m + P) f - \alpha\Pi g]\varphi'' + \{k^4 + k^2[(\sigma R_m + P) f - \alpha\Pi g] + \sigma R_m f (Pf + \alpha\Pi g)\}\varphi = 0,$$
$$f = i(kv - \omega), \quad g = \frac{j^2}{\sigma T},$$
$$\Pi = SQR = \frac{j^{*2}l^2}{\sigma_0\varkappa_0 T_0} = \frac{\sigma_0 B^2 v^{*2} l^2}{\varkappa_0 T_0}. \tag{2.2}$$

The dimensionless number $\alpha\Pi$ may be called the overheating parameter. If Eq. (2.2) admits of finite solutions [9], in the quasi-classical approximation (2.1) we obtain the following dispersion equation:

$$(k_y^2 + k^2)^2 - (k_y^2 + k^2)[-(R_m + P) f + \alpha\Pi] + R_m f (Pf + \alpha\Pi) = 0, \tag{2.3}$$

where k_y, f, R_m, and Π are the values of the slowly varying functions k_y, f, σR_m, and Πg at a specific point y_0. If we require that the roots of Eq. (2.3), quadratic in f, lie in the left halfplane, it is easy to obtain the stability criteria

$$(k_y^2 + k^2)(1 + P/R_m) + \alpha\Pi > 0,$$
$$(k_y^2 + k^2) - \alpha\Pi > 0. \tag{2.4}$$

If $\alpha > 0$, inequality (2.4.1) is automatically satisfied, and we arrive at condition (2.4.2), which was obtained earlier [6] for $R_m \ll 1$. If $\alpha < 0$, inequality (2.4.2) is satisfied identically and the stability criterion is expressed by the weaker condition (2.4.1). Thus, we find that overheating instability is possible when $R_m \gtrsim 1$ and at negative α.

Let us return to the question of the existence of finite solutions of Eq. (2.2). We represent the fourth-order operator on the left of (2.2) as a product of two second-order operators, using their commutivity in a quasi-classical approximation with accuracy to terms of $\sim k_y^{-1}$:

$$D^4\varphi + (r + s)D^2\varphi + rs\varphi = (D^2 + r) \times (D^2 + s)\varphi + O(k_y^{-1})\left(D = \frac{d(\ldots)}{dy}\right). \tag{2.5}$$

Hence, for r and s we have

$$r + s = -2k^2 - [(\sigma R_m + P) f - \alpha\Pi g],$$
$$rs = k^4 + k^2[(\sigma R_m + P) f - \alpha\Pi g] + \sigma R_m f (Pf + \alpha\Pi g). \tag{2.6}$$

In general, formulas (2.6) allow us to reduce the analysis to the study of two second-order equations, but these expressions for r and s are cumbersome. For simplicity, we shall limit ourselves to the case of $R_m \ll P$, which has a fairly wide range of applicability. Then it is easy to show that Eq. (2.2) can be written as

$$(D^2 - k^2)(D^2 - k^2 - Pf + \alpha\Pi g)\varphi = 0. \tag{2.7}$$

Thus, the problem reduces to investigation of the solutions of a Schrödinger equation with the complex potential U + iV

$$\varphi'' - (U + iV)\varphi = 0,$$
$$U = k^2 - \alpha\Pi g + \omega_i P, \quad V = P(kv - \omega_r). \tag{2.8}$$

The initial steady state is symmetric in y, and it can be seen that the real part of the potential has the form of a well and the imaginary part, the form of a hump. If V = 0, finite solutions exist. The finiteness condition is found by joining the solutions that decay at both infinities with the oscillating solutions within the well at the reversal points and it is the phase integral (Bohr quantization condition). It has been shown [9] that the quasi-classical approximation can also be used for equations of the form of (2.8), when the imaginary part of the potential is not zero, if complex reversal points are used. The finiteness condition is the phase integral taken along the line z_c, which connects the reversal points, where $\mathrm{Im}[U(z_c) + iV(z_c)] = 0$, and all values are real if the semiaxes $y \to \pm\infty$ and the lines $|z_c| \to \infty$ lie in a single region bounded by the two anti-Stokes lines [10] To draw qualitative conclusions about stability, however, it is not necessary to find precisely the frequency spectrum from the phase integral, since for each eigenvalue of the frequency ω_n there is a point y_n in the localization region $\varphi(y)$ such that

$$V(y_n, \omega_n, k) = 0,$$
$$\mathrm{Re}\, k_y^2(y_n) + U(y_n, \omega_n, k) = 0. \tag{2.9}$$

If we apply this rule to Eq. (2.8), we find at the boundary of the stability region ($\omega_i = 0$)

$$kv(y_0) = \omega_r, \qquad k_y^2(y_0) + k^2 - \alpha\Pi g(y_0) = 0 \tag{2.10}$$

where y_0 is a point within the flow core. Expression (2.10) coincides with formula (2.4.2). To find the exact instability boundaries and the increments, the problem should be solved by calculating the phase integral at given U(z) and V(z).

We can prove the existence of finite solutions for Eq. (2.8) and derive conditions such as (2.10) by expanding the potential U + iV in series near the axis of the channel and by solving the Schrödinger equation for a harmonic oscillator, as was done, for example, in [11].

3. Now let us examine the effect of thermal conductivity and viscosity on the development of instability. We shall limit ourselves to the case when $R_m \ll 1$. Then, it follows from Eq. (1.8) that

$$\Delta\varphi = -\alpha R_m j\theta / T \tag{3.1}$$

which allows us to eliminate φ from Eq. (1.9). If we let $X = T^{\beta}\theta$, after formal transformations it is not difficult to obtain

$$\chi'' + \Big[-\beta(1 + \beta)(\ln T)'^2 - \beta(\ln T)'' - k^2 + (\gamma - \beta) Q \frac{\eta v'^2}{\varkappa T} + (\alpha - \beta)\Pi \frac{j^2}{\sigma\varkappa T} - iP\frac{kv - \omega}{\varkappa}\Big]\chi = 0. \tag{3.2}$$

The temperature variations in the channel, under the conditions of the problem, are assumed to be relatively small and the number $\beta \sim 1$; therefore, we can drop the first two terms in the brackets in Eq. (3.2).

Thus, we arrive at an equation of the (2.8) type with an additional term governed by viscous energy dissipation.

If α and $\gamma > \beta$, the real part U represents a potential well whose depth is proportional to the overheating parameter $(\alpha - \beta)\Pi$. The depth of the well is further increased if $\alpha > \beta > \gamma$ by a value proportional to $(\beta - \gamma)Q$. Otherwise, when $\alpha < \beta$, the real part of the potential has the form of a hump. The height of the hump is proportional to $(\beta - \alpha)\Pi$, if $\beta > \gamma$ and increases by $\sim(\gamma - \beta)Q$ if $\beta < \gamma$. The existence of finite solutions for Eq. (3.2) when the real part of the potential U is a well was discussed above. If V = 0 and U is a hump, there are no finite solutions. But the presence of the imaginary part V can cause finite solutions to appear even when U is a hump, as was shown in [9] by the example of an equation for a complex "inverted" oscillator. This circumstance, and also the continuous dependence of the solutions of (3.2) on the parameters α, β, and γ, for shortwave perturbations allows us to use quasi-classical approximation (2.1), which gives rise to the stability criterion

$$k_y^2 + k^2 > (\alpha - \beta)\Pi + (\gamma - \beta) Q , \qquad (3.3)$$

where k_y, Π, and Q are the values of the slowly varying functions k_y, $\Pi j^2/\sigma\varkappa T$, and $Q\eta v'^2/\varkappa T$ at some point y_0.

Formula (3.3) makes possible qualitative analysis of the effects of various factors on stability, as well as determination of the possible mechanisms of dissipative instability. The first term on the right of (3.3) is determined by the presence of Joule heating, and the second, by viscous dissipation. The effect of thermal conductivity manifests itself primarily through the index β. Regardless of α and γ, it is apparent from (3.3) that when $\beta > 0$, thermal conductivity promotes flow stability, while when $\beta < 0$, it can be a cause of dissipative instability. For example, when $\alpha = \gamma = 0$, instability can occur, the physical meaning of which consists in the following. With slight overheating of an element of the medium, the thermal-conductivity coefficient and, therefore, the heat flux from that element are reduced, which leads to even greater overheating. The mechanism of viscous dissipative instability is related to the index γ, if $\gamma > 0$. Its physical meaning is similar: with accidental overheating of an element of the fluid, the viscosity coefficient and, consequently, the viscous energy dissipation in the element increase, which results in further heating of the element. It should be noted that these instability mechanisms can also occur in ordinary hydrodynamics. In contrast to these mechanisms, the Leontovich overheating instability associated with the index α is a result of the dependence of conductivity on temperature and Joule energy dissipation.

As can be seen from (3.3), all or part of these instability mechanisms can appear in the problem in question, depending on the relationships between the parameters α, β, and γ. In particular, it is not difficult to see that viscous dissipation can stabilize the instability caused by Joule heating, and vice versa. Different versions may be examined in the specific selection of α, β, and γ.

4. Finally, let us examine overheating instability without assuming small perturbation wavelengths. For simplicity, we shall limit ourselves to the case when $R_m \ll 1$ and ignore the effect of changes in thermal conductivity and viscosity. Then, assuming that $\beta = \gamma = 0$, from (3.2) we obtain

$$L(\theta) = \theta'' + [iP(\omega - kv) - (k^2 - \alpha\Pi g)]\theta = 0. \qquad (4.1)$$

This equation should be solved with homogeneous boundary conditions (1.11). Since Eq. (4.1) is symmetric in y, it is sufficient to consider separately the cases of even and odd perturbations in the interval (0,1). The conditions

$$\theta'(0) = 0, \ \ \theta(1) = 0; \quad \theta(0) = 0, \ \theta(1) = 0 \qquad (4.2)$$

respectively, must be satisfied for even and odd solutions.

We shall use the Galerkin method to solve the eigenvalue problem for (4.1) and (4.2). We take a set of normalized approximating functions $\psi_n(y)$ that satisfy boundary conditions (4.2). If we approximate the solution with the sum

$$\theta = C_1\psi_1 + \ldots + C_p\psi_p \qquad (4.3)$$

from the requirement that the residue of the equation be orthogonal to the approximating functions we arrive at the system of algebraic equations

$$\sum_{n=1}^{p} C_n \int_0^1 \psi_m L(\psi_n)\,dy = 0 \qquad (n, m = 1, 2, \ldots, p). \qquad (4.4)$$

Thus, the problem reduces to investigation of the characteristic equation—the determinant of system (4.4)

$$|L_{mn}| \equiv |-N_{mn}' + iP(\omega N_{mn} - kv_{mn}) - (k^2 N_{mn} - \alpha\Pi g_{mn})| = 0$$

$$\left(N_{mn}' = \int_0^1 \psi_m'\psi_n' dy, \quad N_{mn} = \int_0^1 \psi_m\psi_n dy, \quad v_{mn} = \int_0^1 \psi_m v \psi_n dy\right). \qquad (4.5)$$

Calculation of the first eigenvalue, which determines the boundary of the stability region, is of particular interest. If we let $\omega_i = 0$, if we equate the real and imaginary parts of (4.5) to zero, and if we eliminate ω_r from the two obtained equations, we can, in principle, obtain for fixed P and α an equation of the form

$$F(k^2, \Pi) = 0 . \qquad (4.6)$$

This equation in the plane (k^2, Π) defines a neutral curve that separates the stability region ($\omega_i < 0$) from the instability region ($\omega_i > 0$). It is easy to verify that in first approximation the characteristic equation has the form

$$\omega_r = kv_{11}, \qquad P\omega_i = -N_{11}' - k^2 + \alpha\Pi g_{11} \qquad (4.7)$$

Thus, the neutral curve is a straight line with the slope $(\alpha g_{11})^{-1}$. The accuracy of the first approximation is not great, however, and the dependence of the stability-region boundary on P does not appear in it. Therefore, calculations were made in second approximation. The following sets of approximating functions:

$$\psi_n^{(1)} = N_n(1 - y^{2n}), \quad \psi_n^{(2)} = N_n(\operatorname{ch} n - \operatorname{ch} ny),$$

$$\psi_n^{(3)} = \sqrt{2}\sin n\pi y \quad (n \geqslant 1), \qquad (4.8)$$

where N_n are normalization factors, were used for even and odd perturbations, respectively. In the calculations, the current density $J = 0$, the thermal parameter $N = 1$, and the index $\alpha = 10$ [8] .

A series of neutral curves for various Hartmann numbers M (indicated on the curves) when $P = 1$ for even perturbations is plotted in Fig. 1 on the (k^2, Π) axes. The stability region is located under the neutral curve. When $k \geqslant 10$, the boundary of the stability region is a straight line, in accordance with the results of quasi-classical theory (2.4.2) and (2.10). It is also apparent from Fig. 1 that the slope of the curves and, therefore, the stability region are reduced as the magnetic field is increased.

The effect of thermal conductivity on stability can be seen from Figs. 2 and 3, in which neutral curves are plotted for $M = 0.1$ and 1 and $M = 3, 5$, and 7, respectively, at $P = 1, 10$, and 100 (indicated on the curves). (For convenience, the groups of curves for various M are shifted upward along the axis of the ordinates by a distance of 5.) It should be noted that the curves with $P \ll 1$ are practically the same as the line $P = 1$. At sufficiently high k, the neutral lines for fixed M become straight lines with the same slope, regardless of P. This agrees with the results of quasi-classical theory (2.4.2) and (2.10) (as P increases, the range of values $k \sim P$ increases, in which convergence of the line slopes occurs). It follows from Figs. 2 and 3 that for intermediate and short wavelengths an increase in P results in a certain increase in the stability region. With an increase in M, however, the effect of P is reduced, and the neutral curves begin to merge. The solid lines in Figs. 1-3 show the results of calculations with power functions (4.8.1). For comparison, the dashed lines show the neutral curves calculated with the exponential functions $\psi_n^{(2)}$ at $P = 1$. As can be seen, the results are virtually independent of the choice of the set of approximating functions. The calculations made with functions (4.8.2) showed that the boundary of the stability region is determined by even perturbations. The neutral curves for the odd perturbations are situated in the instability region relative to the even perturbations.

In conclusion, the following fact should be pointed out. For shortwave perturbations, according to (2.10) and (3.2), the phase velocity of the wave is independent of k. Appreciable dispersion occurs in the case of intermediate and long waves. Figure 4 shows a series of curves for $P = 1$ that show (ω/k) as a function of k^2 for neutral oscillations. At high k, the curves become constant, in accordance with (2.10) and (3.2); when $k \leqslant 10$, they have a hump. The figures on the curves indicate the M value, i.e., the wave velocity increases with an increase in the field. If we know (ω/k) at high k from an exact solution and determine the coordinate y_0 using (2.10), we can verify that the slopes of the curves as calculated exactly and with (2.10) coincide.

The author thanks Yu. M. Zolotaikin for programming and performing the calculations.

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24 March 1967 Moscow